Option A Notes:

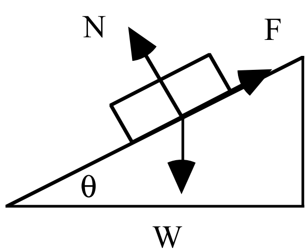
Inclined Planes:

- When we push an object along a surface we need to exert a constant force to keep it moving at constant speed in order to overcome what we call *sliding friction* between the surfaces.
- For two surfaces in contact it turns out that the frictional force is *independent of the area of contact, independent of the relative speed of the two surfaces and is directly proportional to the normal reaction between the two surfaces.*

  $\boxed{F = \mu N}$, where F is the frictional force, N is the normal reaction, and μ is the coefficient of friction between the two surfaces.
- μ depends on the two types of surface in contact.
- We have to determine between static and kinetic (dynamic) friction. Static friction is what is required to get a body moving, and kinetic friction is what is required to keep that object moving.

  $\boxed{F_{fr} = \mu_k N}$ and $\boxed{F_{fr} \leq \mu_s N}$

Example:



— Inclined Plane.

θ is the angle at which the block just starts to slip down the inclined plane.
W sin θ = F = μsN
W cos θ = N

Therefore,

$$\frac{W \sin\theta}{W \cos\theta} = \frac{\mu_s N}{N} \Rightarrow \tan\theta = \mu_s.$$

<u>Fluid Resistance:</u>

- When objects move through a fluid (ie. a liquid or gas), then they also experience a frictional foirce. At low speeds experiment shows that the frictional force for a given fluid and given object is directly proportional to the speed of the object through the fluid. At higher speeds the frictional force becomes proportional to the *square* of the speed.

- This leads to the idea of the terminal velocity. An object falling is subject to a constant force, its weight. The force accelerates the object but as the speed does so does the fluid resistance. Hence it reaches a speed at which the fluid resistance will be equal to the weight and the object will now be in dynamic equilibrium and will continue to fall with constant speed. This is known as the terminal velocity.

Projectile Motion:

- For objects launched at an angle to the horizontal we take vertical and horizontal planes separately:

Vertical Component of Velocity— $\boxed{v_V = v \sin \theta}$

Horizontal Component of Velocity— $\boxed{v_h = v \cos \theta}$

Horizontal Distance Travelled— $\boxed{x = v_h t = (v \cos \theta)t}$

Vertical Distance Travelled— $\boxed{y = (v \sin \theta)t - \frac{1}{2} g t^2}$

Highest Vertical Point— $\boxed{H = \frac{v^2 \sin^2 \theta}{2g}}$

Time to Reach Point— $\boxed{T = \frac{v \sin \theta}{g}}$

Horizontal Range— $\boxed{R = \frac{u^2 \sin \theta}{g}}$

<u>Using Conservation of Energy:</u>

- Using the fact that $E_K + E_P$ = constant at every point in the object's flight, we can solve projectile motion problems this way.

ie. $\frac{1}{2} m v_A^2 = \frac{1}{2} m v_B^2 + mgH = \frac{1}{2} m v_C^2 + mgh.$

A is the starting point (hence no potential energy)
B is the maximum height reached (H is maximum height)
C is some point on the downward motion of the object.

Simple Harmonic Motion:

- If the force acting on a system is directed towards the equilibrium position of the system and is directly proportional to the displacement of the system from equilibrium then the system will execute SHM.

- $\boxed{\omega = \frac{\Delta\theta}{\Delta t}}$ — Angular Velocity.

- $\boxed{x = A \cos \omega t}$ — Displacement of the Mass Oscillating.
- $\boxed{v = -A\ \omega \sin (\omega t)}$ — Instantaneous Velocity
- $\boxed{a = -A\ \omega^2 \cos (\omega t)}$ — Instantaneous Acceleration
- $\boxed{v_{max} = A\omega}$ — Maximum Velocity
- $\boxed{a_{max} = \frac{k}{m} A}$ or $\boxed{|a_{max}| = A\omega^2}$ — Maximum Acceleration
- $\boxed{T = 2\pi \sqrt{\frac{m}{k}}}$ or $\boxed{T = \frac{2\pi}{\omega}}$ — Time period for a harmonic oscillator.

- For Displacement-Time, Velocity-Time Graphs, see IB Text, p. 123.

- When the spring is at its maximum extension the mass is momentarily at rest and all the energy of the oscillator is in the form of elastic potential energy. In general for an extension x, the potential energy is given by:
  $$\boxed{E_{elas} = \frac{1}{2} kx^2}$$
- The maximum kinetic energy of the mass will be when it passes through the equilibrium position. Here the mass has as its maximum velocity $A\omega$ hence the maximum kinetic energy is:
  $$\boxed{KE_{max} = \frac{1}{2} m(A\omega)^2 = \frac{1}{2} mA^2\omega^2}$$
- At any instant, if no energy is lost from the system, the sum of the kinetic energy and potential energy is always constant.
  $KE + PE = \text{constant} = KE_{max} = PE_{max}$.

- Oscillation of a pendulum for small amplitude is also SHM and is given by:
  $$\boxed{T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{l}{g}}}$$

Circular Motion:

- When an object moves with a constant speed v in a circle of radius r from P to Q:
  - $\boxed{\text{Angular Distance} = \Delta\theta = \dfrac{\text{arc length PQ}}{\text{radius } r}}$
  - **Error!**)
  - $\boxed{\text{Displacement} = s = r\theta}$
  - $\boxed{\text{Velocity} = v = r\omega}$
  - $\boxed{\text{Period of Motion} = T = \dfrac{2\pi}{\omega}}$

- Angular Speed of the Earth can be calculated from period of rotation of the earth.

  T = 24 hrs = 24 x 3600s. Therefore $\omega = \dfrac{2\pi}{24 \text{ x } 3600}$.

  Radius of the earth = $6.4 \text{ x } 10^6$ m.

  Therefore, $v = r\omega = 6.4 \text{ x } 10^6 \text{ x } \dfrac{2\pi}{24 \text{ x } 3600} = 470 \text{ ms}^{-1}$.

- $\boxed{\text{Centripetal Acceleration} \Rightarrow a = \dfrac{v^2}{r} \text{ or } a = r\omega^2}$
- $\boxed{\text{Centripetal Force} = ma = \dfrac{mv^2}{r} = mr\omega^2}$
- The effect of the centripetal force is to produce an acceleration towards the centre of the circle. The magnitude of the particle's linear velocity and the magnitud acting on it will determine the circular path that a particular particle describes.
- Situations in which centripetal force arises:
  1. Gravitational Forces.
  2. Frictional Forces between Wheels of a vehicle and the ground.
  3. Magnetic Forces.

<u>In a Vertical Plane</u>:

The tension in a string attached to a mass at the bottom of its path is found by:

$$\boxed{T = mg + \frac{mv^2}{r}}$$

The tension in a string attached to a mass at the top of its path is found by:

$$\boxed{T = mg - \frac{mv^2}{r}}$$

- With circular motion, there is no change in kinetic energy. The speed is constant, and therefore the kinetic energy is constant. Another way to see this is that as the force acts at right angles to the particle then no work is done on the particle by the force.

Universal Gravitation:

- Newton's Law of Universal Gravitation:

  Every particle in the universe attracts every other particle with a force proportional to the product of their masses and inversely proportional to the square of the distance between them.

- This is written mathematically as:

  $$\boxed{F = \frac{G\, m_1 m_2}{r^2}}$$

  G is known as the Universal Gravitational Constant = 6.67 x 10^{11} $\mathrm{Nm^2\,kg^{-2}}$.

- <u>Gravitational Field Strength</u>:

  Any particle exerts a gravitational force on any other particle in the Universe. In this sense we can think of the effect that a particle P produces on other particles without knowing the location of P. We think of a 'field' radiating out from P— ie. the gravitational field. Its strength I at a point X in terms of the force it exerts on P is defined as:

  $\boxed{I = \frac{F}{m}}$ or simply is the gravitational force per unit mass.

  We can see that this in turn is the same as the acceleration. Therefore we can find the acceleration due to gravity using this method:

  $$F = \frac{GMm}{r^2}$$

  Since $I = \frac{F}{m}$,

  $$I = \frac{GM}{r^2}$$

  If we replace the particle of mass M with a sphere of mass M and radius R then relying on the fact that the sphere behaves as a point mass situated at its centre the field strength at its surface is:

  $$I = \frac{GM}{R^2}$$

  If the sphere is the earth:

  $$I = \frac{GM_{\mathrm{e}}}{R_{\mathrm{e}}^2}$$

  But as the field strength is equal to the acceleration, hence it is the acceleration of freefall at the surface of the earth, or the acceleration due to gravity:

$$\boxed{g = \frac{GM_e}{R_e^2}}$$

Hence when we determine g at any point on the Earth we actually measure the gravitational field strength at that point. This hows why g varies with height above the Earth's surface.

Gravitational Potential:

- When we lift an object of mass m to a height h above the surface of the Earth it gains gravitational potential energy of mgh.The expression really gives a difference in potential energy between the value that the object has at the Earth's surface and the value it has at height h. If we choose a point where PE = 0, ie. infinity, this is where the gravitational field strength of any object will be zero.
  The gravitational potential, V at any point in the Earth's field at a distance r from the centre of the Earth, is then found from here:

$$\boxed{V = -\frac{GM_e}{r}}$$

  Gravitational potential is therefore a measure of the amount of work needed to be done to move particles between points in a gravitational field. Its unit is J kg^{-1}. It is negative, so that the potential energy as we move away from the Earth's surface increases until we reach the value of zero at infinity.

Escape Velocity:

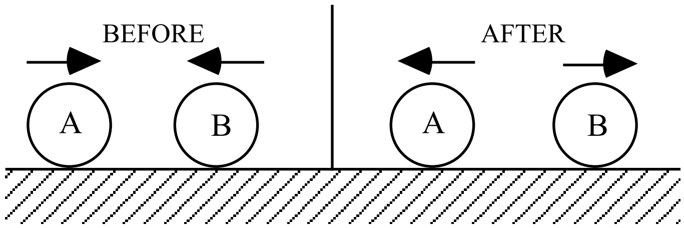
- Escape velocity is 'taking the object to infinity', ie. making it effectively free of the gravitational pull of the Earth or other field:

$$\boxed{v_{\text{escape}} = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2gR_e}}$$

Momentum & Energy:

- Use of Newton's Laws to Deduce Law of Momentum Conservation:

  For a collision of two spheres like thus:



Sphere A has a mass M_A and sphere B has a mass M_B. The velocities before and after, for sphere A and B are v_A, v_B, v_A' and v_B' respectively. During collision F_{AB} is the force that A exerts on B and F_{BA} is the force that B exerts on A. The time for the collision is Δt.

If we consider sphere A then the change of momentum on collision is

$-M_A v_A - M_A v_A'$

From Newton's 2nd Law this change in momentum is equal to the impulse of the force exerted on A. Hence we have

$$F_{BA} = \frac{-M_A v_A - M_A v_A'}{\Delta t}$$

If we consider sphere B, then we have

$$F_{BA} = \frac{-M_B v_B + M_B v_B'}{\Delta t}$$

However from Newton's 3rd Law we have $F_{BA} + F_{BA} = 0$.

Therefore,

Error!) + **Error!**) = 0

$(-M_A v_A - M_A v_A') + (M_B v_B - M_B v_B') = 0.$

This gives:

$M_A v_A - M_B v_B = M_B v_B' - M_A v_A'.$

Hence,

Momentum Before Collision = Momentum After Collision.

Two Dimensional Problems:

- For two balls with velocities $\mathbf{u}_1$ and $\mathbf{u}_2$, where $\mathbf{u}_2 = 0$, and the first ball approaches the second at an angle θ, then they move off at velocities of $\mathbf{v}_1$ and $\mathbf{v}_2$ with angles β and α respectively, the following equations apply:

Parallel to x-axis:

Before:	After:
$\lvert\mathbf{u}_1\rvert \cos\theta$	$\lvert\mathbf{v}_1\rvert \cos\beta + \lvert\mathbf{v}_2\rvert \cos\alpha$

Parallel to y-axis:

Before:	After:
$\lvert\mathbf{u}_1\rvert \sin\theta$	$-\lvert\mathbf{v}_1\rvert \sin\beta + \lvert\mathbf{v}_2\rvert \sin\alpha$

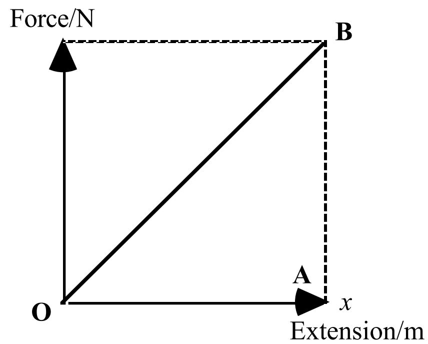
Applying the principle of energy conservation:

$u_1 \cos\theta = v_1 \cos\beta + v_2 \cos\alpha$

$u_1 \sin\theta = -v_1 \cos\beta + v_2 \cos\alpha$

Work Done By a Non-Constant Force and Elastic Potential Energy;

- If we examine a force-extension (displacement) relationship for a spring it appears thus:



- The work done in extending the spring an amount A is the area under the graph, ie. the area OAB.
- If F is the force required to produce an extension A, then this area is equal to $\frac{1}{2}$ FA. But $F = kA$, hence the work done, which is equal to the stored elastic potential energy, is:

  $$\boxed{E_{\text{elas}} = \frac{1}{2}kA^2}$$

  In general, for an extension x, the potential energy is given by:

  $$\boxed{E_{\text{elas}} = \frac{1}{2}kx^2}$$

  The work done by a non-constant force can always be found by computing the area under the force displacement graph for a particular system.